

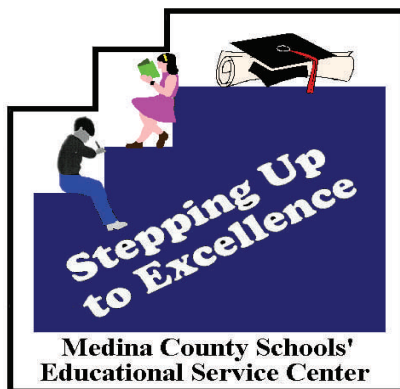
# Medina County Schools

## Math Course of Study

Grades:  
Pre-Kindergarten  
Through Twelve

William J. Koran  
Superintendent

June 2008





Math  
Graded Course of Study  
PreK-12

William J. Koran, Superintendent

Approved by:  
Governing Board of the Medina County Schools'  
Educational Service Center  
2008

Mission Statement

The Medina County Schools'  
Educational Service Center  
will be the leader in providing  
services and products that promote  
excellence in education.

# Acknowledgements

The Medina County Schools' Educational Service Center wishes to acknowledge the contributions to the Math Course of Study made by the following:

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# Introduction

The Graded Course of Study is the approved document which defines the mathematics curriculum to be taught in all Medina County local school districts. This document satisfies the requirements of Ohio law, and is based on the Academic Content Standards developed by the Ohio Department of Education.

Teachers will base their daily plans on this document, thereby assuring each student a complete and comprehensive mathematics education. Through the use of continuous assessment, appropriate adjustments in instruction can be made to intervene with the students who are below grade level and to extend instruction to those students above grade level.

## K-12 Mathematics Philosophy

The Mathematics Academic Content Standards provide a set of clear and rigorous expectations for what all students should know and be able to do by the time they graduate from high school. This K-12 curriculum is designed to insure that all students have the opportunity to become mathematically literate and capable of extending their learning. Students will be confident in their ability to use practical applications solving real life problems.

All students will be challenged by relevant mathematical instruction with the focus on understanding mathematical concepts. Mathematics instruction will include problem solving, reasoning, communicating, and applying mathematics to other curricular areas. All students have the right to learn and develop understandings of significant mathematical concepts. All students must be prepared to pursue a wide range of career options.

The curriculum and the instruction of students must be meaningful. Students must learn to formulate and solve problems using a variety of strategies, check and interpret results, and provide solutions to problems using real-world situations. Teachers must engage in continuous professional development in both the mathematical content area and the effective classroom instruction area. Assessment of learning must be aligned with the Mathematics Graded Course of Study.

# Ohio's K-12 Mathematic Academic Content Standard

## **Number, Number Sense and Operations Standard**

Students demonstrate number sense, including an understanding of number systems and operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

## **Measurement Standard**

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

## **Geometry and Spatial Sense Standard**

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

## **Patterns, Functions and Algebra Standard**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

## **Data Analysis and Probability Standard**

Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

## **Mathematical Processes Standard**

Students use mathematical processes and knowledge to solve problems. Students apply problem-solving and decision-making techniques, and communicate mathematical ideas.

*Note: Mathematical processes are used in all content areas and should be incorporated within instruction and assessment of the content-specific standards, benchmarks and grade-level indicators.*



Medina County Schools'

# Course of Study

For

# Math

Integrated Math I (Cloverleaf, Highland)

Algebra I (Buckeye, Cloverleaf, Highland, MCCC)

Algebra I Academic (Cloverleaf)

Algebra I Basic (Buckeye)

Intervention Review Math (Highland)

Ninth Grade Mathematics (Ohio Academic Content Standards)

June 2008





**STANDARD 2: Measurement**

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

**Ohio Benchmarks  
Grade 9**
**Instructional  
Organization**
**Grade Level Indicators**
**Notes**

<p><b>By the end of the 8-10 program:</b></p> <p>A. Solve increasingly complex non-routine measurement problems and check for reasonableness of results.</p> <p>B. Use formulas to find surface area and volume for specified three-dimensional objects accurate to a specified level of precision.</p> <p>C. Apply indirect measurement techniques, tools and formulas, as appropriate, to find perimeter, circumference and area of circles, triangles, quadrilaterals and composite shapes and to find volume of prisms, cylinders, and pyramids.</p>			
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**STANDARD 2: Measurement (Cont.)**

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

Ohio Benchmarks  
Grade 9Instructional  
Organization

## Grade Level Indicators

## Notes

Ohio Benchmarks Grade 9	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 8-10 program:</b></p> <p>D. Use proportional reasoning and apply indirect measurement techniques, including right triangle trigonometry and properties of similar triangles, to solve problems involving measurements and rates.</p>	M.2.D.9.1 <i>Measurement Units</i>	1. Convert rates within the same measurement system; e.g., miles per hour to feet per second; kilometers per hour to meters per second.	
	M.2.D.9.2 <i>Use measurement Techniques and Tools</i>	2. Use unit analysis to check computations involving measurement.	
	M.2.D.9.3	3. Use the ratio of lengths in similar two-dimensional figures or three-dimensional objects to calculate the ratio of their areas or volumes respectively.	
	M.2.D.9.4	4. Use scale drawings and right triangle trigonometry to solve problems that include unknown distances and angle measures.	
	M.2.D.9.5	5. Solve problems involving unit conversion for situations involving distances, areas, volumes and rates within the same measurement system.	
E. Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision.			

**STANDARD 2: Measurement (Cont.)**

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

Ohio Benchmarks  
Grade 9

Instructional  
Organization

Grade Level Indicators

Notes

<p><b>By the end of the 8-10 program:</b></p> <p>F. Write and solve real-world, multi-step problems involving money, elapsed time and temperature, and verify reasonableness of solutions.</p>			
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**STANDARD 3: Geometry and Spatial Sense**

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Ohio Benchmarks  
Grade 9

Instructional  
Organization

Grade Level Indicators

Notes

<p><b>By the end of the 8-10 program:</b></p> <p>A. Formally define geometric figures.</p> <p>B. Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.</p> <p>C. Recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines, and parallel lines.</p> <p>D. Use coordinate geometry to represent and examine the properties of geometric figures.</p> <p>E. Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools, such as straightedge, compass and technology.</p>			
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**STANDARD 3: Geometry and Spatial Sense (Cont.)**

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Ohio Benchmarks  
Grade 9Instructional  
Organization

## Grade Level Indicators

## Notes

By the end of the 8-10 program:			
F. Represent and model transformations in a coordinate plane and describe the results.			
G. Prove or disprove conjectures and solve problems involving two- and three-dimensional objects represented within a coordinate system.	M.3.G.9.3 <i>Visualization and Geometric Models</i>	3. Analyze two-dimensional figures in a coordinate plane; e.g., use slope and distance formulas to show that a quadrilateral is a parallelogram.	
H. Establish the validity of conjectures about geometric objects, their properties and relationships by counter-example, inductive and deductive reasoning, and critiquing arguments made by others.			
I. Use right triangle trigonometric relationships to determine lengths and angle measures.	M.3.I.9.1 <i>Characteristics and Properties</i>	1. Define the basic trigonometric ratios in right triangles: sine, cosine and tangent.	
	M.3.I.9.2	2. Apply proportions and right triangle trigonometric ratios to solve problems involving missing lengths and angle sizes in similar figures.	

**STANDARD 4: Patterns, Functions and Algebra**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 9	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 8-10 program:</b></p> <p>A. Generalize and explain patterns and sequences in order to find the next term and the <math>n</math>th term.</p> <p>B. Identify and classify functions as linear or nonlinear, and contrast their properties using tables, graphs or equations.</p> <p>C. Translate information from one representation (words, table, graph or equation) to another representation of a relation or function.</p>	<p>M.4.A.9.2 <i>Use Patterns, Relations and Functions</i></p> <p>M.4.B.9.1 <i>Use Patterns, Relations and Functions</i></p> <p>M.4.B.9.3</p> <p>M.4.C.9.2</p>	<p>2. Generalize patterns using functions or relationships (linear, quadratic and exponential), and freely translate among tabular, graphical and symbolic representations.</p> <p>1. Define function with ordered pairs in which each domain element is assigned exactly one range element.</p> <p>3. Describe problem situations (linear, quadratic and exponential) by using tabular, graphical and symbolic representations.</p> <p>2. Generalize patterns using functions or relationships (linear, quadratic and exponential), and freely translate among tabular, graphical and symbolic representations.</p>	

**STANDARD 4: Patterns, Functions and Algebra (Cont.)**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 9	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 8-10 program:</b></p> <p>D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.</p> <p>E. Analyze and compare functions and their graphs using attributes, such as rates of change, intercepts and zeros.</p>	<p>M.4.D.9.7 <i>Use Algebraic Expressions</i></p> <p>M.4.D.9.11</p> <p>M.4.D.9.12</p> <p>M.4.E.9.4 <i>Use Patterns, Relations and Functions</i></p> <p>M.4.E.9.5</p>	<p>7. Use formulas to solve problems involving exponential growth and decay.</p> <p>11. Add, subtract, multiply and divide monomials and polynomials (division of polynomials by monomials only).</p> <p>12. Simplify rational expressions by eliminating common factors and applying properties of integer exponents.</p> <p>4. Demonstrate the relationship among zeros of a function, roots of equations, and solutions of equations graphically and in words.</p> <p>5. Describe and compare characteristics of the following families of functions: linear, quadratic and exponential functions; e.g., general shape, number of roots, domain, range, rate of change, maximum or minimum.</p>	

**STANDARD 4: Patterns, Functions and Algebra (Cont.)**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 9	Instructional Organization	Grade Level Indicators	Notes
<b>By the end of the 8-10 program:</b>			
F. Solve and graph linear equations and inequalities.	M.4.F.9.6 <i>Use Algebraic Representations</i>	6. Write and use equivalent forms of equations and inequalities in problem situations; e.g., changing a linear equation to the slope-intercept form.	
G. Solve quadratic equations with real roots by graphing, formula and factoring.	M.4.F.9.8	8. Find linear equations that represent lines that pass through a given set of ordered pairs, and find linear equations that represent lines parallel or perpendicular to a given line through a specific point.	
H. Solve systems of linear equations involving two variables graphically and symbolically.	M.4.G.9.10	10. Solve quadratic equations with real roots by factoring, graphing, using the quadratic formula and with technology.	
	M.4.H.9.9 <i>Use Algebraic Representations</i>	9. Solve and interpret the meaning of 2 by 2 systems of linear equations graphically, by substitution and by elimination, with and without technology.	

**STANDARD 4: Patterns, Functions and Algebra (Cont.)**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 9	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 8-10 program:</b></p> <p>I. Model and solve problem situations involving direct and inverse variation.</p> <p>J. Describe and interpret rates of change from graphical and numerical data.</p>	<p>M.4.I.9.13 <i>Analyze Change</i></p> <p>M.4.I.9.14</p> <p>M.4.J.9.15</p>	<p>13. Model and solve problems involving direct and inverse variation using proportional reasoning.</p> <p>14. Describe the relationship between slope and the graph of a direct variation and inverse variation.</p> <p>15. Describe how a change in the value of a constant in a linear or quadratic equation affects the related graphs.</p>	

**STANDARD 5: Data Analysis and Probability**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 9	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 8-10 program:</b></p> <p>A. Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatter plots, measures of center and variability.</p> <p>B. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose.</p> <p>C. Compare the characteristics of the mean, median and mode for a given set of data, and explain which measure of center best represents the data.</p>	<p>M.5.A.9.1 <i>Data Collection</i></p> <p>M.5.A.9.2</p> <p>M.5.A.9.3 <i>Statistical Methods</i></p>	<ol style="list-style-type: none"> <li>1. Classify data as univariate (single variable) or bivariate (two variables) and as quantitative (measurement) or qualitative (categorical) data.</li> <li>2. Create a scatterplot for a set of bivariate data, sketch the line of best fit, and interpret the slope of the line of best fit.</li> <li>3. Analyze and interpret frequency distributions based on spread, symmetry, skewness, clusters and outliers.</li> </ol>	

**STANDARD 5: Data Analysis and Probability (Cont.)**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 9	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 8-10 program:</b></p> <p>D. Find, use and interpret measures of center and spread, such as mean and quartiles, and use those measures to compare and draw conclusions about sets of data.</p> <p>E. Evaluate the validity of claims and predictions that are based on data by examining the appropriateness of the data collection and analysis.</p> <p>F. Construct convincing arguments based on analysis of data and interpretation of graphs.</p> <p>G. Describe sampling methods and analyze the effects of method chosen on how well the resulting sample represents the population.</p>	<p>M.5.E.9.4</p> <p>M.5.F.9.6</p> <p>M.5.G.9.5</p>	<p>4. Describe and compare various types of studies (survey, observation, experiment), and identify possible misuses of statistical data.</p> <p>6. Make inferences about relationships in bivariate data, and recognize the difference between evidence of relationship (correlation) and causation.</p> <p>5. Describe characteristics and limitations of sampling methods, and analyze the effects of random versus biased sampling; e.g., determine and justify whether the sample is likely to be representative of the population.</p>	

**STANDARD 5: Data Analysis and Probability (Cont.)**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 9	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 8-10 program:</b></p> <p>H. Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.</p> <p>I. Design an experiment to test a theoretical probability, and record and explain results.</p> <p>J. Compute probabilities of compound events, independent events, and simple dependent events.</p> <p>K. Make predictions based on theoretical probabilities and experimental results.</p>	<p>M.5.H.9.7 <i>Probability</i></p> <p>M.5.I.9.8</p> <p>M.5.J.9.9</p> <p>M.5.K.9.10</p>	<p>7. Use counting techniques and the Fundamental Counting principle to determine the total number of possible outcomes for mathematical situations.</p> <p>8. Describe, create and analyze a sample space and use it to calculate probability.</p> <p>9. Identify situations involving independent and dependent events, and explain differences between and common misconceptions about probabilities associated with those events.</p> <p>10. Use theoretical and experimental probability, including simulations or random numbers, to estimate probabilities and to solve problems dealing with uncertainty; e.g., compound events, independent events, simple dependent events.</p>	



**STANDARD 6: Mathematical Processes (Cont.)**

Students use mathematical processes and knowledge to solve problems. Students apply problem-solving and decision-making techniques, and communicate mathematical ideas.

Ohio Benchmarks Grade 9	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 8-10 program:</b></p> <p>D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.</p> <p>E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.</p> <p>F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.</p> <p>G. Write clearly and coherently about mathematical thinking and ideas.</p> <p>H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.</p>	<p>M.6.D.9</p> <p>M.6.E.9</p> <p>M.6.F.9</p> <p>M.6.G.9</p> <p>M.6.H.9</p>		



Medina County Schools'

# Course of Study

For

# Math

Glossary

June 2008

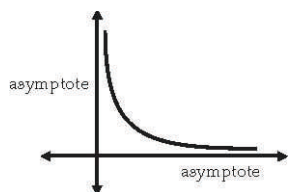
**absolute error** The absolute value of the difference between the measured value of a quantity and its true value.

**acute** An angle whose measure is greater than  $0^\circ$  and less than  $90^\circ$ .

**algorithm** A procedure or series of steps used to solve a problem.

**associative property** The result of an operation on real numbers will be unchanged due to grouping; e.g., for addition,  $(a + b) + c = a + (b + c)$  or for multiplication,  $a(bc) = (ab)c$ .

**asymptote** A straight line that a curve approaches but never touches. For example,



**biased sampling** A sample that overrepresents or underrepresents part of the population.

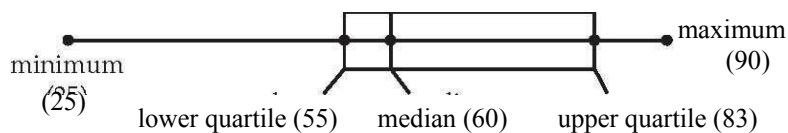
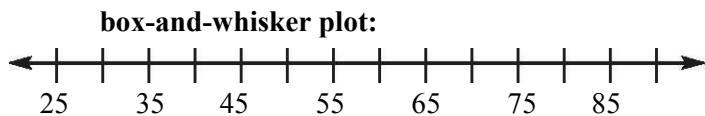
**bivariate data** Data or events described using two variables.

**box-and-whisker plot** A diagram that shows pictorially the *median* and *measures of spread* (upper and lower *interquartile ranges* and the *range*) for one set of data. For example,

**Box-and-whisker plot data:**

35	25	90	60	45
40	58	90	90	55
60	55	80	90	60
55	60	85	75	60
56	55	75	80	90

The number of days students in Mr. Jones' homeroom spent studying for the ACT exam.



- causation** The relationship between two *variables* where a change in one *variable* affects the outcome of the other *variable*.
- categorical data** Data that can be classified by type; e.g., color, types of dogs. These types of data are typically represented using bar chart, pie charts or pictographs.
- central angle** An angle whose vertex is the center of a circle and is in the same plane as the circle.
- 
- Central angle
- coefficient** The numeric factor in a term; e.g., the number 3 in the term  $3x^2y$  is the coefficient or in the term  $a^3b$ , 1 is the coefficient.
- Combination** A selection of a group of items or events from a set without regard to order; e.g., the number of 3-piece outfits from the set of clothes in the closet.
- common factor** A number, *polynomial* or quantity that evenly divides into two or more mathematical expressions.
- common referents** Something that is familiar that can be used to relate to another **referents** thing that is not familiar; e.g., the width of a finger is a centimeter.
- Commutative property** The order of the objects in an operation can be changed with out **property** affecting the results; e.g., for addition,  $a + b = b + a$  or for multiplication,  $ab = ba$ .
- compatible numbers** Numbers that go together easily, usually related by pairing in the basic **numbers** facts; use of compatible numbers generally gives an approximate result; e.g.,  $473 \div 6 \approx 80$ .
- Compensatory numbers** Compensatory numbers are used to adjust numbers in a computation after use of *compatible numbers*; e.g.,  $23 + 18 = 23 + 20 = 43$ . Since two was added to increase 18 to 20 as compatible numbers, two will be subtracted from 43 to compensate for the change. Therefore, two is the compensatory number.
- complementary events** Two or more *mutually exclusive events* that together cover all possible **events** outcomes. The sum of the probabilities of complementary events is 1.
- compound events** Combining two or more separate events or outcomes and considering events it as one single event or outcome.
- conditional probability** The probability of an event occurring given that another event has already occurred. For example, What is the probability that the total of two dice will be greater than 8 given that the first die is a 6?

- congruent** Having exactly the same size and shape.
- continuous data** Data that can be assigned an infinite number of values between whole numbers, the assigned values are approximated; e.g., the size of the apples on an apple tree is continuous data. See *discrete data* for a counterexample.
- Coordinate plane** A plane determined by the intersection of two perpendicular number lines in which any point can be located.
- correlation** The relation between two sets of data, a positive or direct correlation exists when both sets vary in the same direction (both sets decrease); a negative or inverse correlation exists when one set of data increases as the other decreases.
- correlation** A measure of the *correlation* between two *variables* or sets of data.
- coefficient** The value of the correlation coefficient,  $r$ , is always  $-1 < r < 1$ , where 1 is a perfect positive correlation, 0 is no correlation, and -1 is a perfect negative correlation.
- covariants** Varying with another variable quantity in a manner that leaves a specified relationship unchanged.
- decomposing** The process of breaking a number into smaller units to simplify problem solving; e.g., 15 can be  $10 + 5$  or 10 can be  $6 + 4$ .
- deductive reasoning** Use logic to arrive at a conclusion from a given premise.
- dependent events** A statement or *probability* for one event affects a statement or *probability for another event*.
- descriptive statistics** To gather and describe data using *probability*, statistical methods and concepts like graphs and *measures of center*.
- dispersion** How data is spread out around some central point.
- distribution** The distribution of a set of data is a graph or table showing how many pieces of data there are in each class, or of each type.
- distributive property** The product of a number and the sum (or difference) of two numbers is equal to the sum (or difference) of the two products; e.g.,  $7(30 + 5) = (7 \cdot 30) + (7 \cdot 5)$  or  $a(b-c) = ab - ac$ .
- equation** A statement that shows two mathematical expressions that are equal to each other.
- equiangular** In a given shape, all angles have the same measure.
- equilateral** In a given shape, all sides have the same length.
- equivalent** Two items that have the same value.

**experimental probability** The probability based on a series of trials. The experimental probability,  $P$ , can be found using the following equation:  $P(\text{event}) = \frac{\text{\# of trials w/favorable outcomes}}{\text{\# of trials in experiment}}$

**experimental results** The outcome as a result of a probability experiment or test. These outcomes are sometimes called actual results.

**expressions** Any combination of variables, numbers, and symbols (excluding the equality and inequality symbols).

**extrema** A term that refers to maximum and minimum values.

**factoring** Rewriting a mathematical expression as a product of factors.

**frequency distribution** A collection of data that represents the number of times a set of numbers, items or events have occurred.

**frequency table** A table that shows how often each item, number, or range of numbers occurs in a set of data.

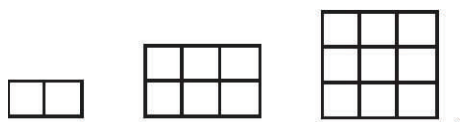
**front-end estimation** Using the leading, or left-most, digits to make an estimate quickly and easily. After making an initial estimate using front-end digits, an adjustment can be made to refine the estimate; e.g., Using front-end estimation to estimate the sum of 594, 32, and 221, an initial estimate would be  $5 + 0 + 2$  hundreds or 700. An adjustment can be made by grouping the tens and ones (about  $100 + 50$  or 150 more) and adding to get an adjusted estimate of 850.

**function** A mathematical relationship between two variables, an independent *variable* and a *dependent variable*, where every value of the independent variable corresponds to exactly one value of the dependent value.

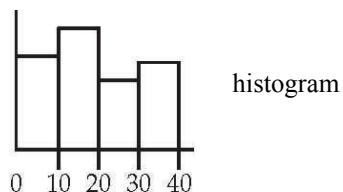
**Fundamental Counting Principle** The principle which states that all possible outcomes in a sample space can be found by multiplying the number of ways each event can occur.

**geometric patterns** A sequence or series, where each term can be found by multiplying the previous term by a constant factor, sometimes referred to as a common ratio.

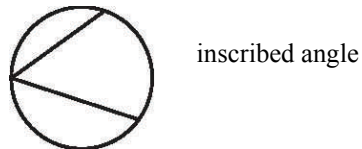
**growing patterns** Patterns that involve a progression. For example,



**histogram** A graph that uses bars to show the frequency of data within equal intervals.



**inscribed angle** An angle whose vertex is on a circle and whose sides are chords of the circle.



**measures of center** Numbers that provide information about cluster and average of a collection of data.

**mean** The sum of a set of numbers divided by the number of elements in the set.

**mode** The number or object that appears most frequently in a set of numbers or objects.

**median** The middle number or item in a set of numbers or objects arranged from least to greatest, or the mean of the two middle numbers when the set has two middle numbers.

**measures of spread or variability** A term used to refer to how much numbers are spread, varied or dispersed in a set of data.

**range** The difference between the greatest and the least numbers in a set of data.

**quartile** In conjunction with the median, the quartiles divide the set of data into four groups of equal size.

**interquartile range** The difference between the upper quartile range and the lower quartile.

**median** See measures of center.

**minor arc** An arc that is less than a semicircle or  $180^\circ$ .

**mode** See measures of center.

**monomials** An algebraic expression which is a product of constants and variables.

**multiplicative patterns** Number patterns with relationships between consecutive numbers involving multiplication.

**Mutually exclusive events** Two events that cannot occur at the same time.

**nonlinear** A sequence of values that increase in a manner other than linear.

**outlier** A data point in a sample widely separated from the main cluster of points in the sample.

**parallel lines** Lines in the same plane that do not cross, the distance between the lines is constant.

**permutations** Possible orders or arrangements of a set of events or items.

**perpendicular lines** Lines that intersect at one point forming  $90^\circ$ .

**polygon** A closed figure formed from line segments that meet only at their endpoints.

**polynomials** The sum of monomials; e.g.,  $2a^2 + 4a - 5$ .

**precision** To determine the size of the unit to be used.

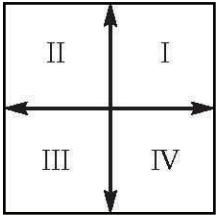
**prime factorization** The expression of a number as the product of prime factors; e.g., the prime factorization of 18 is  $2 \cdot 3 \cdot 3$ .

**probability** The chance of an event occurring. The probability of an event is equal to the number of favorable outcomes divided by the number of possible outcomes.

**probability distribution** The set of random data and the probabilities associated with that data.

**proportion** An equation showing that two ratios are equal.

**quadrants** The two axes of a coordinate system divide the plane into four separate sections known as quadrants. These are identified as the first, second, third, and fourth quadrants.



**qualitative data** Data that can be assigned qualities or categories. They are non-numerical data.

**quantitative data** Data that are numerical. The data can be *discrete* or *continuous*.

**random sample** A *sample* in which every event has an equal chance of selection and each event is chosen by a random process.

**random sampling** A random *sample* is a sample that has been chosen by a process of random selection so that it models the characteristics of the population it is supposed to represent as closely as possible.

**random variable** A variable that takes any of a range of values that cannot be predicted with certainty.

**rate of change** A relationship such as distance over time, often described by using a slope.

**rational expressions** Fractions whose numerators and denominators are polynomials; e.g.,  $\frac{n^2 - 3n}{2}$ .

**rational numbers** Any number that can be written in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are integers and  $b \neq 0$ .

**rectangular arrays** An arrangement of things or data in rows and columns.

**Recursive function** A function defined in terms of the repeated application of a number of simpler functions to their own values.

**reflection** See transformation.

**relative error** The error or uncertainty in a measurement expressed as a fraction of the true value.

**right** Relating to  $90^\circ$ ; e.g., a right angle measures  $90^\circ$ , a right triangle has only one right angle.

**roots of equations** A value that will satisfy the equation which has been formed by putting an expression, containing one *variable*, equal to zero.

**rotation** See transformation.

**sample** A set of data taken from a larger set used to create or test theories about the data as a whole.

**sample space** A list of all possible outcomes of an activity.

**sampling method** The process used to collect data; e.g., see random sampling.

**scientific notation** A form of writing numbers as the product of a power of 10 and a decimal number greater than or equal to 1 and less than 10; e.g., 8,924,000 is written as  $8.924 \times 10^6$ .

**sequence** An ordered set of objects or numbers.

**series** Sum of a finite or infinite sequence of terms.

**simple event** A subset of the *sample space* that contains only one outcome that cannot be broken down into a simpler, more basic outcome.

**standard deviation** The measure of the *dispersion* of a distribution is equal to the square root of the *variance*.

**stem-and-leaf plot** A frequency diagram which displays the actual data together with its frequency, by using a part of the value of each piece of data to fix the class or group (the stem), while the remainder of the value is actually listed (the leaves). For example,

**Stem-and-leaf plot data:** Coach Smith's last 30 basketball game scores for the 7th grade Wildcats.

50	65	70	35	40	57	66	65	70	35
29	33	44	56	66	60	44	50	58	46
67	78	79	47	35	35	44	57	60	57

Stem-and-leaf plot

Stem	Leaves
2	9
3	3 5 5 5 6
4	0 4 4 4 6 7
5	0 0 6 7 7 7 8
6	0 0 5 5 6 6 7
7	0 0 8 9

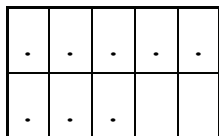
Key: 4 | 6 represents a score of 46.

**successive approximation** To find the approximate value of a quantity by starting from a first estimate and then deriving from each approximation another that is more accurate.

**symbolic form** To represent something using numbers and symbols.

**target population** The set from which a sample will be selected.

**tens frame** A physical model that represents the structure of the number system's place value; e.g., the following diagram represents the number eight using a tens frame.



tens frame

**terms** The quantities in an algebraic equation that are linked to each other by means of + or - signs.

**theoretical probability** Identifying, using mathematical expectations, the number of ways an event could happen compared to all the events that could happen.

**transcendental function** *Functions* that are not algebraic; e.g., trigonometric functions.

**transformation** An operation that creates an image from an original figure, or preimage.

**reflection** A *transformation* that results in a mirror image of the original shape.

**rotation** A rotation is a *transformation* about a fixed point such that every point in the object turns through the same angle relative to that fixed point.

**translation** A *transformation* in which an image is formed by moving every point on a figure the same distance in the same direction.

**dilation** A *transformation* that preserves the shape of a figure, but allows the size to change.

**translation** See transformation.

**two-dimensional figures** A shape that has two dimensions, usually described in terms of length and breadth, or length and height.

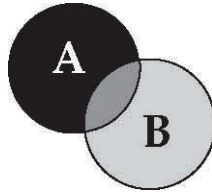
**univariate data** Having one *variable*.

**variable** A changing quantity, usually a letter in an algebraic equation or expression, that might have one of a range of possible values.

**variance** A measure of the *dispersion* of the *distribution* of a *random variable*.

**variants** *Variables.*

**Venn Diagrams** A diagram that is used to show relationships between sets.



**zeros of a function** The solutions of a *function* or the x-intercepts.

