

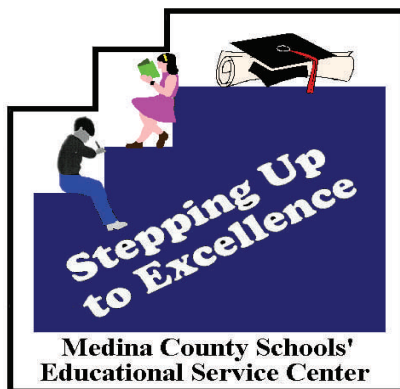
# Medina County Schools

## Math Course of Study

Grades:  
Pre-Kindergarten  
Through Twelve

William J. Koran  
Superintendent

June 2008





Math  
Graded Course of Study  
PreK-12

William J. Koran, Superintendent

Approved by:  
Governing Board of the Medina County Schools'  
Educational Service Center  
2008

Mission Statement

The Medina County Schools'  
Educational Service Center  
will be the leader in providing  
services and products that promote  
excellence in education.

# Acknowledgements

The Medina County Schools' Educational Service Center wishes to acknowledge the contributions to the Math Course of Study made by the following:

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# Introduction

The Graded Course of Study is the approved document which defines the mathematics curriculum to be taught in all Medina County local school districts. This document satisfies the requirements of Ohio law, and is based on the Academic Content Standards developed by the Ohio Department of Education.

Teachers will base their daily plans on this document, thereby assuring each student a complete and comprehensive mathematics education. Through the use of continuous assessment, appropriate adjustments in instruction can be made to intervene with the students who are below grade level and to extend instruction to those students above grade level.

## K-12 Mathematics Philosophy

The Mathematics Academic Content Standards provide a set of clear and rigorous expectations for what all students should know and be able to do by the time they graduate from high school. This K-12 curriculum is designed to insure that all students have the opportunity to become mathematically literate and capable of extending their learning. Students will be confident in their ability to use practical applications solving real life problems.

All students will be challenged by relevant mathematical instruction with the focus on understanding mathematical concepts. Mathematics instruction will include problem solving, reasoning, communicating, and applying mathematics to other curricular areas. All students have the right to learn and develop understandings of significant mathematical concepts. All students must be prepared to pursue a wide range of career options.

The curriculum and the instruction of students must be meaningful. Students must learn to formulate and solve problems using a variety of strategies, check and interpret results, and provide solutions to problems using real-world situations. Teachers must engage in continuous professional development in both the mathematical content area and the effective classroom instruction area. Assessment of learning must be aligned with the Mathematics Graded Course of Study.

# Ohio's K-12 Mathematic Academic Content Standard

## **Number, Number Sense and Operations Standard**

Students demonstrate number sense, including an understanding of number systems and operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

## **Measurement Standard**

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

## **Geometry and Spatial Sense Standard**

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

## **Patterns, Functions and Algebra Standard**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

## **Data Analysis and Probability Standard**

Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

## **Mathematical Processes Standard**

Students use mathematical processes and knowledge to solve problems. Students apply problem-solving and decision-making techniques, and communicate mathematical ideas.

*Note: Mathematical processes are used in all content areas and should be incorporated within instruction and assessment of the content-specific standards, benchmarks and grade-level indicators.*



Medina County Schools'

# Course of Study

For

# Math

Algebra II (Buckeye, Highland, MCCC)  
Algebra II Honors (Buckeye, Highland)  
Technical Math (MCCC)  
Transition Math (Buckeye)  
Integrated Math/Integrated Math III (MCCC)  
Integrated III (Highland)  
Transition to College Math (Highland)  
Eleventh Grade Mathematics (Ohio Academic Content Standards)

June 2008

## STANDARD 1: Number, Number Sense and Operations

Grade 11

Students demonstrate number sense, including an understanding of number systems and operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Ohio Benchmarks  
Grades 11

Instructional  
Organization

Grade Level Indicators

Notes

<p><b>By the end of the 11-12 program:</b></p> <p>A. Demonstrate that vectors and matrices are systems having some of the same properties of the real number system.</p> <p>B. Develop an understanding of properties of and representations for addition and multiplication of vectors and matrices.</p>	<p>M.1.A.11.1 <i>Number and Number Systems</i></p> <p>M.1.A.11.2</p> <p>M.1.B.11.1 <i>Number and Number Systems</i></p> <p>M.1.B.11.2</p> <p>M.1.B.11.5</p>	<p>1. Determine what properties hold for matrix addition and matrix multiplication; e.g., use examples to show addition is commutative and when multiplication is not commutative.</p> <p>2. Determine what properties hold for vector addition and multiplication, and for scalar multiplication.</p> <p>1. Determine what properties hold for matrix addition and matrix multiplication; e.g., use examples to show addition is commutative and when multiplication is not commutative.</p> <p>2. Determine what properties hold for vector addition and multiplication, and for scalar multiplication.</p> <p>5. Model using the coordinate plane, vector addition and scalar multiplication.</p>	
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## STANDARD 1: Number, Number Sense and Operations (Cont.)

Grade 11

Students demonstrate number sense, including an understanding of number systems and operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Ohio Benchmarks Grades 11	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 11-12 program:</b></p> <p>C. Apply factorials and exponents, including fractional exponents, to solve practical problems.</p> <p>D. Demonstrate fluency in operations with real numbers, vectors and matrices, using mental computation or paper and pencil calculations for simple cases, and technology for more complicated cases.</p> <p>E. Represent and compute with complex numbers.</p>	M.1.C.10.3	3. Use factorial notation and computations to represent and solve problem situations involving arrangements. <b>(Grade 10)</b>	
	M.1.C.11.8	8. Use fractional and negative exponents as optional ways of representing and finding solutions for problem situations; e.g., $(27^{2/3})^2 = (27^{1/3})^2 = 9$ .	
	M.1.D.11.4 <i>Meaning of Operations</i>	4. Use matrices to represent given information in a problem situation.	
	M.1.D.11.6 <i>Computation and Estimation</i>	6. Compute sums, differences and products of matrices using paper and pencil calculations for simple cases, and technology for more complicated cases.	
	M.1.D.11.9	9. Use vector addition and scalar multiplication to solve problems.	
	M.1.E.11.3	3. Represent complex numbers on the complex plane.	
	M.1.E.11.7	7. Compute sums, differences, products and quotients of complex numbers.	

**STANDARD 2: Measurement**

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

**Ohio Benchmarks  
Grade 11**

**Instructional  
Organization**

**Grade Level Indicators**

**Notes**

By the end of the 11-12 program:			
A. Explain differences among accuracy, precision and error, and describe how each of those can affect solutions in measurement situations.	M.2.A.10.1 <i>Use Measurement Techniques and Tools</i>	1. <i>Explain how a small error in measurement may lead to a large error in calculated results. (Grade 10)</i>	
	M.2.A.10.2	2. <i>Calculate relative error. (Grade 10)</i>	
	M.2.A.10.3	3. <i>Explain the difference between absolute error and relative error in measurement. (Grade 10)</i>	
	M.2.A.10.4	4. <i>Give examples of how the same absolute error can be problematic in one situation but not in another; e.g., compare “accurate to the nearest foot” when measuring the height of a person versus when measuring the height of a mountain. (Grade 10)</i>	
B. Apply various measurement scales to describe phenomena and solve problems.	M.2.A.11.1 <i>Measurement Units</i>	1. Determine the number of significant digits in a measurement.	
	M.2.B.11.2	2. Use radian and degree angle measures to solve problems and perform conversions as needed.	

**STANDARD 2: Measurement (Cont.)**

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

Ohio Benchmarks  
Grade 11Instructional  
Organization

## Grade Level Indicators

## Notes

Ohio Benchmarks Grade 11	Instructional Organization	Grade Level Indicators	Notes
<b>By the end of the 11-12 program:</b>			
C. Estimate and compute areas and volume in increasingly complex problem situations.	M.2.C.11.3 <i>Use Measurement Techniques and Tools</i>	3. Derive a formula for the surface area of a cone as a function of its slant height and the circumference of its base.	
	M.2.C.11.4	4. Calculate distances, areas, surface areas and volumes of composite three-dimensional objects to a specified number of significant digits.	
D. Solve problem situations involving derived measurements; e.g., density, acceleration.	M.2.D.11.5	5. Solve real-world problems involving area, surface area, volume and density to a specified degree of precision.	

**STANDARD 3: Geometry and Spatial Sense**

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Ohio Benchmarks  
Grade 11Instructional  
Organization

## Grade Level Indicators

## Notes

<p><b>By the end of the 11-12 program:</b></p> <p>A. Use trigonometric relationships to verify determine solutions in problem situations.</p> <p>B. Represent transformations within a coordinate system using vectors and matrices.</p> <p><i>Note: This is an extension of the following benchmarks in grades 8-10 for more complex figures.</i></p> <p>A. Formally define geometric figures.</p> <p>D. Use coordinate geometry to represent and examine the properties of geometric figures.</p> <p>E. Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools, such as straightedge, compass and technology.</p>	<p>M.3.A.11.4 <i>Transformation and Symmetry</i></p> <p>M.3.B.11.1 <i>Spatial Relationships</i></p> <p>M.3.B.11.2 <i>Transformations and Symmetry</i></p> <p>M.3.B.11.3</p> <p>M.3.A. .5 <i>Visualization and Geometric Models</i></p>	<p>4. Use trigonometric relationships to determine lengths and angle measures; i.e., Law of Sines and Law of Cosines.</p> <p>1. Use polar coordinates to specify locations on a plane.</p> <p>2. Represent translations using vectors.</p> <p>3. Describe multiplication of a vector and a scalar graphically and algebraically, and apply to problem situations.</p> <p>5. Identify, sketch and classify the cross sections of three-dimensional objects.</p>	
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**STANDARD 3: Geometry and Spatial Sense (Cont.)**

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Ohio Benchmarks  
Grade 11

Instructional  
Organization

Grade Level Indicators

Notes

<p><b>By the end of the 11-12 program:</b></p> <p><i>Note: This is an extension of benchmark H in grades 11-12 in Mathematical Processes.</i></p> <p>H. Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.</p>			
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**STANDARD 4: Patterns, Functions and Algebra**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 11	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 11-12 program:</b></p> <p>A. Analyze functions by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.</p>	<p>M.4.A.11.3 <i>Use Patterns, Relations and Functions</i></p> <p>M.4.A.11.4</p> <p>M.4.A.11.5</p> <p>M.4.A.11.6 <i>Use Algebraic Representations</i></p> <p>M.4.A.11.10</p> <p>M.4.A.11.11 <i>Analyze Change</i></p>	<p>3. Describe and compare the characteristics of the following families of functions: quadratics with complex roots, polynomials of any degree, logarithms, and rational functions; e.g., general shape, number of roots, domain and range, asymptotic behavior.</p> <p>4. Identify the maximum and minimum points of polynomial, rational and trigonometric functions graphically and with technology.</p> <p>5. Identify families of functions with graphs that have rotation symmetry or reflection symmetry about the <math>y</math>-axis, <math>x</math>-axis or <math>y = x</math>.</p> <p>6. Represent the inverse of a function symbolically and graphically as a reflection about <math>y = x</math>.</p> <p>10. Describe the characteristics of the graphs of conic sections.</p> <p>11. Describe how a change in the value of a constant in an exponential, logarithmic or radical equation affects the graph of the equation.</p>	

**STANDARD 4: Patterns, Functions and Algebra (Cont.)**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 11	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 11-12 program:</b></p> <p>B. Use the quadratic formula to solve quadratic equations that have complex roots.</p> <p>C. Use recursive functions to model and solve problems; e.g., home mortgages, annuities.</p> <p>D. Apply algebraic methods to represent and generalize problem situations involving vectors and matrices.</p>	<p>M.4.B.11.8 <i>Use Algebraic Representatives</i></p> <p>M.4.C.11.1 <i>Use Patterns, Relations and Functions</i></p> <p>M.4.C.11.2</p> <p>M.4.D.11.7 <i>Use Algebraic Representatives</i></p> <p>M.4.D.11.9</p>	<p>8. Solve equations involving radical expressions and complex roots.</p> <p>1. Identify and describe problem situations involving an iterative process that can be represented as a recursive function; e.g., compound interest.</p> <p>2. Translate a recursive function into a closed form expression or formula for the <math>n</math>th term to solve a problem situation involving an iterative process; e.g., find the value of an annuity after 7 years.</p> <p>7. Model and solve problems with matrices and vectors.</p> <p>9. Solve 3 by 3 systems of linear equations by elimination and using technology, and interpret graphically what the solution means (a point, line, plane, or no solution).</p>	

**STANDARD 5: Data Analysis and Probability**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 11	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 11-12 program:</b></p> <p>A. Create and analyze tabular and graphical displays of data using appropriate tools, including spreadsheets and graphing calculators.</p>	<p>M.5.A.11.4 <i>Statistical Methods</i></p> <p>M.5.A.11.5</p> <p>M.5.A.11.7</p> <p>M.5.A.11.8</p> <p>M.5.A.11.10 <i>Probability</i></p>	<p>4. Create a scatterplot of bivariate data, identify trends, and find a function to model the data.</p> <p>5. Use technology to find the Least Squares Regression Line, the regression coefficient, and the correlation coefficient for bivariate data with a linear trend, and interpret each of these statistics in the context of the problem situation.</p> <p>7. Describe the standard normal curve and its general properties, and answer questions dealing with data assumed to be normal.</p> <p>8. Analyze and interpret univariate and bivariate data to identify patterns, note trends, draw conclusions, and make predictions.</p> <p>10. Understand and use the concept of random variable, and compute and interpret the expected value for a random variable in simple cases.</p>	

**STANDARD 5: Data Analysis and Probability (Cont.)**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 11	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 11-12 program:</b></p> <p>B. Use descriptive statistics to analyze and summarize data, including measures of center, dispersion, correlation and variability.</p>	<p>M.5.B.11.3 <i>Statistical Methods</i></p> <p>M.5.B.11.5</p> <p>M.5.B.11.6</p> <p>M.5.B.11.8</p>	<p>3. Describe how a linear transformation of univariate data affects range, mean, mode, and median.</p> <p>5. Use technology to find the Least Squares Regression Line, the regression coefficient, and the correlation coefficient for bivariate data with a linear trend, and interpret each of these statistics in the context of the problem situation.</p> <p>6. Use technology to compute the standard deviation for a set of data, and interpret standard deviation in relation to the context or problem situation.</p> <p>8. Analyze and interpret univariate and bivariate data to identify patterns, note trends, draw conclusions, and make predictions.</p>	

**STANDARD 5: Data Analysis and Probability (Cont.)**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 11	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 11-12 program:</b></p> <p>C. Design and perform a statistical experiment, simulation or study; collect and interpret data; and use descriptive statistics to communicate and support predictions and conclusions.</p>	<p>M.5.C.11.1 <i>Data Collection</i></p> <p>M.5.C.11.2</p> <p>M.5.C.11.9 <i>Statistical Methods</i></p>	<p>1. Design a statistical experiment, survey or study for a problem; collect data for the problem; and interpret the data with appropriate graphical displays, descriptive statistics, concepts of variability, causation, correlation and standard deviation.</p> <p>2. Describe the role of randomization in a well -designed study, especially as compared to a convenience sample, and the generalization of results from each.</p> <p>9. Evaluate validity of results of a study based on characteristics of the study design, including sampling method, summary statistics and data analysis techniques.</p>	

**STANDARD 5: Data Analysis and Probability (Cont.)**

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Ohio Benchmarks Grade 11	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 11-12 program:</b></p> <p>D. Connect statistical techniques to applications in workplace and consumer situations.</p>	<p>M.5.D.11.1 <i>Data Collection</i></p> <p>M.5.D.11.2</p> <p>M.5.D.11.9</p> <p>M.5.D.11.11 <i>Probability</i></p>	<p>1. Design a statistical experiment, survey or study for a problem; collect data for the problem; and interpret the data with appropriate graphical displays, descriptive statistics, concepts of variability, causation, correlation and standard deviation.</p> <p>2. Describe the role of randomization in a well -designed study, especially as compared to a convenience sample, and the generalization of results from each.</p> <p>9. Evaluate validity of results of a study based on characteristics of the study design, including sampling method, summary statistics and data analysis techniques.</p> <p>11. Examine statements and decisions involving risk; e.g., insurance rates and medical decisions.</p>	

**STANDARD 6: Mathematical Processes**

Students use mathematical processes and knowledge to solve problems. Students apply problem-solving and decision-making techniques, and communicate mathematical ideas.

Ohio Benchmarks Grade 11	Instructional Organization	Grade Level Indicators	Notes
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<p><b>By the end of the 11-12 program:</b></p> <p>A. Construct algorithms for multi-step and non-routine problems.</p> <p>B. Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.</p> <p>C. Assess the adequacy and reliability of information available to solve a problem.</p> <p>D. Select and use various types of reasoning and methods of proof.</p> <p>E. Evaluate a mathematical argument and use reasoning and logic to judge its validity.</p> <p>F. Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.</p>	<p>M.6.A.11</p> <p>M.6.B.11</p> <p>M.6.C.11</p> <p>M.6.D.11</p> <p>M.6.E.11</p> <p>M.6.F.11</p>	<p>Note: Mathematical processes are used within all of the content standards and should be incorporated within the instruction and assessment of the benchmarks and grade-level indicators.</p>	
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**STANDARD 6: Mathematical Processes (Cont.)**

Students use mathematical processes and knowledge to solve problems. Students apply problem-solving and decision-making techniques, and communicate mathematical ideas.

Ohio Benchmarks Grade 11	Instructional Organization	Grade Level Indicators	Notes
<p><b>By the end of the 11-12 program:</b></p> <p>G. Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.</p> <p>H. Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.</p> <p>I. Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.</p> <p>J. Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model, and validation to original problem situation.</p>	<p>M.6.G.11</p> <p>M.6.H.11</p> <p>M.6.I.11</p> <p>M.6.J.11</p>		



Medina County Schools'

# Course of Study

For

# Math

Glossary

June 2008

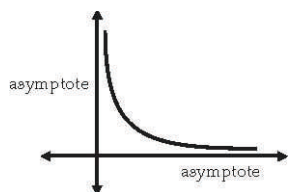
**absolute error** The absolute value of the difference between the measured value of a quantity and its true value.

**acute** An angle whose measure is greater than  $0^\circ$  and less than  $90^\circ$ .

**algorithm** A procedure or series of steps used to solve a problem.

**associative property** The result of an operation on real numbers will be unchanged due to grouping; e.g., for addition,  $(a + b) + c = a + (b + c)$  or for multiplication,  $a(bc) = (ab)c$ .

**asymptote** A straight line that a curve approaches but never touches. For example,



**biased sampling** A sample that overrepresents or underrepresents part of the population.

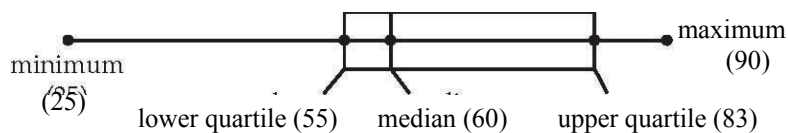
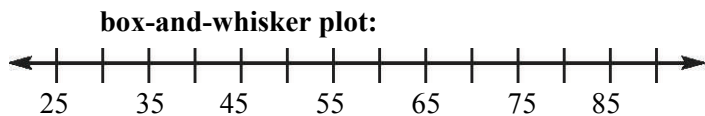
**bivariate data** Data or events described using two variables.

**box-and-whisker plot** A diagram that shows pictorially the *median* and *measures of spread* (upper and lower *interquartile ranges* and the *range*) for one set of data. For example,

**Box-and-whisker plot data:**

35	25	90	60	45
40	58	90	90	55
60	55	80	90	60
55	60	85	75	60
56	55	75	80	90

The number of days students in Mr. Jones' homeroom spent studying for the ACT exam.



- causation** The relationship between two *variables* where a change in one *variable* affects the outcome of the other *variable*.
- categorical data** Data that can be classified by type; e.g., color, types of dogs. These types of data are typically represented using bar chart, pie charts or pictographs.
- central angle** An angle whose vertex is the center of a circle and is in the same plane as the circle.
- 
- Central angle
- coefficient** The numeric factor in a term; e.g., the number 3 in the term  $3x^2y$  is the coefficient or in the term  $a^3b$ , 1 is the coefficient.
- Combination** A selection of a group of items or events from a set without regard to order; e.g., the number of 3-piece outfits from the set of clothes in the closet.
- common factor** A number, *polynomial* or quantity that evenly divides into two or more mathematical expressions.
- common referents** Something that is familiar that can be used to relate to another **referents** thing that is not familiar; e.g., the width of a finger is a centimeter.
- Commutative property** The order of the objects in an operation can be changed with out **property** affecting the results; e.g., for addition,  $a + b = b + a$  or for multiplication,  $ab = ba$ .
- compatible numbers** Numbers that go together easily, usually related by pairing in the basic **numbers** facts; use of compatible numbers generally gives an approximate result; e.g.,  $473 \div 6 \approx 80$ .
- Compensatory numbers** Compensatory numbers are used to adjust numbers in a computation after use of *compatible numbers*; e.g.,  $23 + 18 = 23 + 20 = 43$ . Since two was added to increase 18 to 20 as compatible numbers, two will be subtracted from 43 to compensate for the change. Therefore, two is the compensatory number.
- complementary events** Two or more *mutually exclusive events* that together cover all possible **events** outcomes. The sum of the probabilities of complementary events is 1.
- compound events** Combining two or more separate events or outcomes and considering events it as one single event or outcome.
- conditional probability** The probability of an event occurring given that another event has already occurred. For example, What is the probability that the total of two dice will be greater than 8 given that the first die is a 6?

- congruent** Having exactly the same size and shape.
- continuous data** Data that can be assigned an infinite number of values between whole numbers, the assigned values are approximated; e.g., the size of the apples on an apple tree is continuous data. See *discrete data* for a counterexample.
- Coordinate plane** A plane determined by the intersection of two perpendicular number lines in which any point can be located.
- correlation** The relation between two sets of data, a positive or direct correlation exists when both sets vary in the same direction (both sets decrease); a negative or inverse correlation exists when one set of data increases as the other decreases.
- correlation** A measure of the *correlation* between two *variables* or sets of data.
- coefficient** The value of the correlation coefficient,  $r$ , is always  $-1 < r < 1$ , where 1 is a perfect positive correlation, 0 is no correlation, and -1 is a perfect negative correlation.
- covariants** Varying with another variable quantity in a manner that leaves a specified relationship unchanged.
- decomposing** The process of breaking a number into smaller units to simplify problem solving; e.g., 15 can be  $10 + 5$  or 10 can be  $6 + 4$ .
- deductive reasoning** Use logic to arrive at a conclusion from a given premise.
- dependent events** A statement or *probability* for one event affects a statement or *probability for another event*.
- descriptive statistics** To gather and describe data using *probability*, statistical methods and concepts like graphs and *measures of center*.
- dispersion** How data is spread out around some central point.
- distribution** The distribution of a set of data is a graph or table showing how many pieces of data there are in each class, or of each type.
- distributive property** The product of a number and the sum (or difference) of two numbers is equal to the sum (or difference) of the two products; e.g.,  $7(30 + 5) = (7 \cdot 30) + (7 \cdot 5)$  or  $a(b-c) = ab - ac$ .
- equation** A statement that shows two mathematical expressions that are equal to each other.
- equiangular** In a given shape, all angles have the same measure.
- equilateral** In a given shape, all sides have the same length.
- equivalent** Two items that have the same value.

**experimental probability** The probability based on a series of trials. The experimental probability,  $P$ , can be found using the following equation:  $P(\text{event}) = \frac{\text{\# of trials w/favorable outcomes}}{\text{\# of trials in experiment}}$

**experimental results** The outcome as a result of a probability experiment or test. These outcomes are sometimes called actual results.

**expressions** Any combination of variables, numbers, and symbols (excluding the equality and inequality symbols).

**extrema** A term that refers to maximum and minimum values.

**factoring** Rewriting a mathematical expression as a product of factors.

**frequency distribution** A collection of data that represents the number of times a set of numbers, items or events have occurred.

**frequency table** A table that shows how often each item, number, or range of numbers occurs in a set of data.

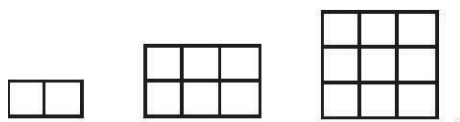
**front-end estimation** Using the leading, or left-most, digits to make an estimate quickly and easily. After making an initial estimate using front-end digits, an adjustment can be made to refine the estimate; e.g., Using front-end estimation to estimate the sum of 594, 32, and 221, an initial estimate would be  $5 + 0 + 2$  hundreds or 700. An adjustment can be made by grouping the tens and ones (about  $100 + 50$  or 150 more) and adding to get an adjusted estimate of 850.

**function** A mathematical relationship between two variables, an independent *variable* and a *dependent variable*, where every value of the independent variable corresponds to exactly one value of the dependent value.

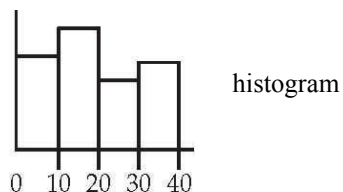
**Fundamental Counting Principle** The principle which states that all possible outcomes in a sample space can be found by multiplying the number of ways each event can occur.

**geometric patterns** A sequence or series, where each term can be found by multiplying the previous term by a constant factor, sometimes referred to as a common ratio.

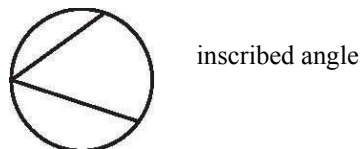
**growing patterns** Patterns that involve a progression. For example,



**histogram** A graph that uses bars to show the frequency of data within equal intervals.



**inscribed angle** An angle whose vertex is on a circle and whose sides are chords of the circle.



**measures of center** Numbers that provide information about cluster and average of a collection of data.

**mean** The sum of a set of numbers divided by the number of elements in the set.

**mode** The number or object that appears most frequently in a set of numbers or objects.

**median** The middle number or item in a set of numbers or objects arranged from least to greatest, or the mean of the two middle numbers when the set has two middle numbers.

**measures of spread or variability** A term used to refer to how much numbers are spread, varied or dispersed in a set of data.

**range** The difference between the greatest and the least numbers in a set of data.

**quartile** In conjunction with the median, the quartiles divide the set of data into four groups of equal size.

**interquartile range** The difference between the upper quartile range and the lower quartile.

**median** See measures of center.

**minor arc** An arc that is less than a semicircle or  $180^\circ$ .

**mode** See measures of center.

**monomials** An algebraic expression which is a product of constants and variables.

**multiplicative patterns** Number patterns with relationships between consecutive numbers involving multiplication.

**Mutually exclusive events** Two events that cannot occur at the same time.

**nonlinear** A sequence of values that increase in a manner other than linear.

**outlier** A data point in a sample widely separated from the main cluster of points in the sample.

**parallel lines** Lines in the same plane that do not cross, the distance between the lines is constant.

**permutations** Possible orders or arrangements of a set of events or items.

**perpendicular lines** Lines that intersect at one point forming  $90^\circ$ .

**polygon** A closed figure formed from line segments that meet only at their endpoints.

**polynomials** The sum of monomials; e.g.,  $2a^2 + 4a - 5$ .

**precision** To determine the size of the unit to be used.

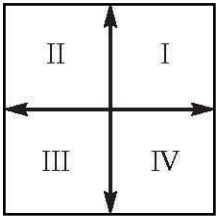
**prime factorization** The expression of a number as the product of prime factors; e.g., the prime factorization of 18 is  $2 \cdot 3 \cdot 3$ .

**probability** The chance of an event occurring. The probability of an event is equal to the number of favorable outcomes divided by the number of possible outcomes.

**probability distribution** The set of random data and the probabilities associated with that data.

**proportion** An equation showing that two ratios are equal.

**quadrants** The two axes of a coordinate system divide the plane into four separate sections known as quadrants. These are identified as the first, second, third, and fourth quadrants.



**qualitative data** Data that can be assigned qualities or categories. They are non-numerical data.

**quantitative data** Data that are numerical. The data can be *discrete* or *continuous*.

**random sample** A *sample* in which every event has an equal chance of selection and each event is chosen by a random process.

**random sampling** A random *sample* is a sample that has been chosen by a process of random selection so that it models the characteristics of the population it is supposed to represent as closely as possible.

**random variable** A variable that takes any of a range of values that cannot be predicted with certainty.

**rate of change** A relationship such as distance over time, often described by using a slope.

**rational expressions** Fractions whose numerators and denominators are polynomials; e.g.,  $\frac{n^2 - 3n}{2}$ .

**rational numbers** Any number that can be written in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are integers and  $b \neq 0$ .

**rectangular arrays** An arrangement of things or data in rows and columns.

**Recursive function** A function defined in terms of the repeated application of a number of simpler functions to their own values.

**reflection** See transformation.

**relative error** The error or uncertainty in a measurement expressed as a fraction of the true value.

**right** Relating to  $90^\circ$ ; e.g., a right angle measures  $90^\circ$ , a right triangle has only one right angle.

**roots of equations** A value that will satisfy the equation which has been formed by putting an expression, containing one *variable*, equal to zero.

**rotation** See transformation.

**sample** A set of data taken from a larger set used to create or test theories about the data as a whole.

**sample space** A list of all possible outcomes of an activity.

**sampling method** The process used to collect data; e.g., see random sampling.

**scientific notation** A form of writing numbers as the product of a power of 10 and a decimal number greater than or equal to 1 and less than 10; e.g., 8,924,000 is written as  $8.924 \times 10^6$ .

**sequence** An ordered set of objects or numbers.

**series** Sum of a finite or infinite sequence of terms.

**simple event** A subset of the *sample space* that contains only one outcome that cannot be broken down into a simpler, more basic outcome.

**standard deviation** The measure of the *dispersion* of a distribution is equal to the square root of the *variance*.

**stem-and-leaf plot** A frequency diagram which displays the actual data together with its frequency, by using a part of the value of each piece of data to fix the class or group (the stem), while the remainder of the value is actually listed (the leaves). For example,

**Stem-and-leaf plot data:** Coach Smith's last 30 basketball game scores for the 7th grade Wildcats.

50	65	70	35	40	57	66	65	70	35
29	33	44	56	66	60	44	50	58	46
67	78	79	47	35	35	44	57	60	57

Stem-and-leaf plot

Stem	Leaves
2	9
3	3 5 5 5 6
4	0 4 4 4 6 7
5	0 0 6 7 7 7 8
6	0 0 5 5 6 6 7
7	0 0 8 9

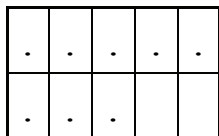
Key: 4 | 6 represents a score of 46.

**successive approximation** To find the approximate value of a quantity by starting from a first estimate and then deriving from each approximation another that is more accurate.

**symbolic form** To represent something using numbers and symbols.

**target population** The set from which a sample will be selected.

**tens frame** A physical model that represents the structure of the number system's place value; e.g., the following diagram represents the number eight using a tens frame.



tens frame

**terms** The quantities in an algebraic equation that are linked to each other by means of + or - signs.

**theoretical probability** Identifying, using mathematical expectations, the number of ways an event could happen compared to all the events that could happen.

**transcendental function** *Functions* that are not algebraic; e.g., trigonometric functions.

**transformation** An operation that creates an image from an original figure, or preimage.

**reflection** A *transformation* that results in a mirror image of the original shape.

**rotation** A rotation is a *transformation* about a fixed point such that every point in the object turns through the same angle relative to that fixed point.

**translation** A *transformation* in which an image is formed by moving every point on a figure the same distance in the same direction.

**dilation** A *transformation* that preserves the shape of a figure, but allows the size to change.

**translation** See transformation.

**two-dimensional figures** A shape that has two dimensions, usually described in terms of length and breadth, or length and height.

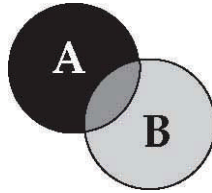
**univariate data** Having one *variable*.

**variable** A changing quantity, usually a letter in an algebraic equation or expression, that might have one of a range of possible values.

**variance** A measure of the *dispersion* of the *distribution* of a *random variable*.

**variants** *Variables.*

**Venn Diagrams** A diagram that is used to show relationships between sets.



**zeros of a function** The solutions of a *function* or the x-intercepts.

